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# Influence of condensation on the stability of a liquid film moving under the effect of gravity and turbulent vapor flow

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**Abstract**

The linear stability of condensate film flowing on an inclined isothermal plate under action of gravity and turbulent vapor flow was the subject of study. The cases of cocurrent and countercurrent flow of two phases were considered at an arbitrary inclination of the plane. The first part of this work deals with stationary film flow. The impact of vapor flow on the film is described by a given shear stress on the interface with account for the transverse mass flux due to phase transition. The integral method gives the analytical solution for distribution of film thickness along the plane (with and without account for film inertia) at different inclination angles. The second part of paper deals with linear stability of stationary film flow. The fluctuation of shear stress on the surface was calculated using the quasilaminar model. The two-wave equation for film thickness with phase transition and dispersion formulas were derived. The results of effect of condensation on film stability are presented for a wide range of flow parameters.


## 1. Introduction

The cooperative motion of gas flow and liquid film takes places in different apparatuses operating in the fields of power engineering and chemical technology. The instability of interface surface has a tremendous impact on heat and mass transfer, primarily due to formation of waves, turbulization, droplet separation, and formation of dry spots. Many of research papers were devoted to problem of linear and nonlinear stability of a free falling liquid film [1–8]. However, the systematic study of this problem with consideration for phase transition was initiated only in 1970s. The papers [2–6] deal with study of conditions for wave formation on a surface of the falling film and they prove that, although the mass flux to interface has a serious effect stability, the critical Reynolds numbers are too small and almost everywhere the condensate film is unstable. The conclusion is that condensation process gain stability to film flow and evaporation destabilize the film flow. The integral method [7] was applied to study of stability of vertical condensate film as function of dimensionless parameters that characterize the physical properties of liquid and vapor. Two different mechanisms of mass flux impact on interface were described. The first mechanism (stabilization of film flow) takes place due to reduction in the film kinetic energy because of attaching of additional mass of condensate. The second mechanism works due to destabilizing effect of reactive force on the interface this effect is visible only at high intensities of phase transition.

The process of wave formation in cooperative gas-liquid flow was studied in a less extend. The existence

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**Nomenclature**

| | |
|---|---|
| $C_f$ | drag coefficient |
| $c_p$ | specific heat of liquid |
| $g$ | gravitational acceleration |
| $h$ | film thickness |
| $H$ | dimensionless perturbations of thickness |
| $j$ | condensation mass flow |
| $k$ | wave number |
| $L$ | latent heat |
| $p$ | pressure |
| $p'$ | dimensionless perturbations of pressure |
| $Q$ | dimensionless perturbations of flow rate |
| $q$ | flow rate per width unit of the film |
| $t$ | time |
| $T_s$ | vapor saturation temperature |
| $T_W$ | wall temperature |
| $T$ | liquid temperature |
| $u_s$ | film surface velocity |
| $V$ | vapor velocity |
| $u, v$ | velocities components |
| $x, y$ | coordinates |

*Dimensionless groups*

$Ku = L/c_p(T_s - T_W)$ Kutateladze number
$Re = gh_m^3/3\nu^2$ film Reynolds number
$We = \sigma/\rho h_m u_m^2$ Weber number
$R_v = V/u_m$, $R_m = V(3/\nu g)^{1/3}$ vapor velocity
$r_m = C_f \rho_v R_m |R_m| / 2\rho$ shear stress

$Fi = \sigma^3/\rho^3 g \nu^4$ film number

*Greek symbols*

| | |
|---|---|
| $\rho, \rho_v$ | liquid and vapor density |
| $\lambda$ | liquid thermal conductivity |
| $\nu$ | liquid kinematic viscosity |
| $\mu, \mu_v$ | liquid and vapor dynamic viscosity |
| $\tau_s$ | shear stress at the interface |
| $\tau_W$ | shear stress at the wall |
| $\tau_f = C_f \cdot \rho_v V\|V\|/2$ | contribution to the shear stress caused by friction of moving vapor |
| $\varepsilon = \dfrac{1}{Ku \cdot Pr}$ | condensation intensity |
| $\sigma$ | surface tension |
| $\theta$ | inclination angle to horizon |
| $\tau'$ | dimensionless perturbations of shear stress |
| $\tau = 3 \cdot \tau_f/\rho g h_m$ | dimensionless shear stress |
| $\eta = y/h$ | dimensionless coordinate |

*Subscripts*

| | |
|---|---|
| W | wall |
| m | scale |
| f | friction due to the moving vapor |
| v | vapor |
| s | on the interface |
| n | neutral |
| max | maximal growth |
| o | undisturbed |

of shear stress on film surface due to gas motion is significant in formation of waves (even without phase transition). In [9], solutions of Orr–Sommerfeld equations for gas and liquid flows were applied for study of wave formation in film flow with regard to phase transition and shear stress on the film surface at different inclination angles of the plane. The case of cocurrent vapor flow was considered only. The problem of wave generation on the interface in the conjugated statement creates big mathematical difficulties. However, this problem can be simplified for many cases if we use several assumptions about properties of gas and liquid. This approach takes the surface of liquid as rigid and steady. Then one can calculate independently the gas motion along a wavy surface. The impact of gas on growth of small disturbances in the film thickness is expressed through the given amplitudes of fluctuation for tangential and normal stress on the film. The amplitudes of stress fluctuation were calculated in [10] from solution of the problem of gas flow over the wavy surface based on the quasilaminar model of turbulent gas flow [11,12]. Those results were used in [13,14] for study of stability of vertical and horizontal gas-film flow based upon Orr–Sommerfeld equations. In [15], the quasilaminar model of turbulent gas flow was employed in the integral method for analysis of stability of joint gas-film flow; a two-wave equation for film thickness was derived. The dispersion relations (obtained from this two-wave equation) describe two different wavy modes. One of these modes may produce instability, and another is responsible for decay. In [16], integral method and Orr–Sommerfeld equations were applied for analysis of stability of gas-film flow without phase transition; the results of these two approaches have been compared.

The objective of this paper is study of impact of condensation of moving vapor on linear stability of film flow using the approaches of integral method and quasilaminar model.

## 2. Problem statement

Let us introduce the Cartesian coordinate system, with axis $Ox$ directed along the plane and axis $Oy$ normal to the plane, which is inclined at angle $\theta$ to horizon (Fig. 1); let us consider the joint motion of vapor and condensate film at the following assumptions:

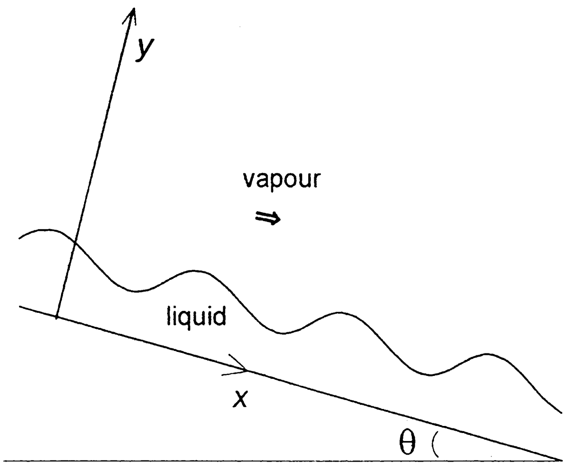


Fig. 1. Scheme of flow.

1) Plate temperature $T_W = \text{const}$.
2) The interface temperature is equal to the temperature of saturated vapor $T_s = \text{const}$.
3) The condensate film brings the main contribution to thermal resistance.
4) The vapor fills all space above the film; the vapor velocity is $V = \text{const}$, which is much smaller than the sonic velocity; we neglect the vapor pressure gradient.
5) The ratio of thickness of vapor boundary layer to the film thickness is small, i.e., $\frac{\rho_v \cdot \mu}{\rho \cdot \mu} \ll 1$.
   In this case the film flow is considered independently of the vapor flow. The influence of vapor flow on the film is taken into account through mass flux and through normal and tangential shear stress on the interface.
6) We neglect the contribution of the reactive force caused by phase transition into the normal stress.

Using this framework for problem statement we can present (as it was done in [17]) the shear stress on the film surface in the form $\tau_s = \tau_f + j \cdot (V - u_s)$. Here $\tau_f = C_f \cdot \rho_v V|V|/2$ is the contribution to the shear stress caused by friction of moving vapor (the drag coefficient $C_f = \text{const}$ must be given); $j \cdot (V - u_s)$ is the contribution of mass flux through interface into the shear stress, $j = \left| -\frac{\lambda}{L} \cdot \frac{\partial T}{\partial y} \right|_{y=h}$ is the mass flux through the area unit of interface (caused by phase transition), $\lambda$ is the thermal conductivity, and $L$ is the latent heat of phase transition.

## 3. Integral equation of momentum for a film with phase transition

Let us consider the motion equation for a film in the approximation of boundary layer, with assumption of low curvature of the film surface [18]:

$$\begin{aligned} &\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \\ &\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\sin\theta - \frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} \end{aligned} \tag{1}$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial y} - g\cos\theta \tag{2}$$

Here the velocity components $u, v$ have to satisfy the boundary condition $u|_{y=0} = 0, v|_{y=0} = 0$ on the plane. In Eq. (2), the vertical velocity component $v$ is assumed infinitely small. By integrating the second Eq. (1) over the film thickness and transforming it with use of the first Eq. (1) the integrals in the left part (as in [18]), we obtain the momentum equation in the form:

$$\begin{aligned} &\frac{\partial q}{\partial t} + \frac{\partial J}{\partial x} - u_s \cdot \left(\frac{\partial h}{\partial t} + u_s\frac{\partial h}{\partial x} - v_s\right) \\ &\quad = \frac{\tau_s - \tau_W}{\rho} - \frac{1}{\rho}\int_0^h \frac{\partial p}{\partial x}\,dy + g \cdot h \cdot \sin\theta \end{aligned} \tag{3}$$

Here $J = \int_0^h u^2\,dy$, $q = \int_0^h u\,dy$ is the liquid flow rate, $\tau_W = \mu\frac{\partial u}{\partial y}|_{y=0}$ is the shear stress on the plate,

$$\tau_s = \mu\frac{\partial u}{\partial y}|_{y=h} = \tau_f + j \cdot (V - u_s) \tag{4}$$

is the shear stress on the film surface. The pressure in the film can be found from the Eq. (2) with account of the boundary condition on the film surface: $p|_{y=h} = p_s - \sigma \cdot h_{xx}$

$$p = p_s + \rho g\cos\theta \cdot (h - y) - \sigma \cdot h_{xx} \tag{5}$$

Here $p_s$ is the vapor pressure. The following kinematic condition takes place on the film surface:

$$\frac{\partial h}{\partial t} + u_s\frac{\partial h}{\partial x} - v_s = j/\rho \tag{6}$$

With account for (4)–(6), the momentum equation for a film takes the form:

$$\begin{aligned} &\frac{\partial q}{\partial t} + \frac{\partial J}{\partial x} \\ &\quad = \frac{\tau_v - \tau_W}{\rho} - \frac{h}{\rho}\frac{\partial p_s}{\partial x} + gh \cdot \left(\sin\theta - \cos\theta\frac{\partial h}{\partial x}\right) + \frac{\sigma h}{\rho}\frac{\partial^3 h}{\partial x^3} \end{aligned} \tag{7}$$

Here

$$\tau_v = \tau_f + j \cdot V \tag{8}$$

The further simplification of (7) is possible only if we will prescribe the velocity profile in the film (not self-similar):

$$\begin{aligned} &u/u_s = (2 - A) \cdot \eta + (A - 1) \cdot \eta^2, \quad \text{where} \\ &\quad A = \tau_s h/\mu u_s, \quad \eta = y/h \end{aligned} \tag{9}$$

and temperature profile in the film: $T = T_W + (T_s - T_W) \cdot \eta$ (linear approximation), which satisfies the boundary conditions $T|_{y=0} = T_W, T|_{y=h} = T_s$.

Then $j = \left| -\frac{\lambda}{L} \cdot \frac{\partial T}{\partial y} \right|_{y=h} = \frac{1}{Ku \cdot Pr} \cdot \frac{\mu}{h} \cdot$, where $Ku \cdot = L/c_p(T_s - T_W)$ is the Kutateladze criterion, and $Pr = \nu/a$ is the Prandtl number. One can easily formulate from (9) the velocity on the film surface through the parameters of flow rate and shear stress $\tau_v$:

$$u_s = (3q/2h + \tau_v h/4\mu)/(1 + \varepsilon/4) \tag{10}$$

Here $\varepsilon = \frac{j \cdot h}{\mu} = \frac{1}{Ku \cdot Pr}$ is the parameter describing the intensity of phase transition. For most of liquids, $Pr \cong 1 - 10$, so at $Ku \gg 1$ we have parameter $\varepsilon \ll 1$. The exception is liquid metals, since they have $Pr \cong 10^{-2}$. For this reason, $\varepsilon \approx 1$ even at $Ku \cong 100$. We can find from (9) the shear stress on the plate

$$\tau_W = K_1 \cdot \left( \frac{3\mu q}{h^2} - \frac{\tau_v}{2} \right) \tag{11}$$

Putting (11) into (7), we finally obtain the equation for film momentum:

$$\frac{\partial q}{\partial t} + \frac{\partial J}{\partial x} = K_2 \cdot \frac{3\tau_v}{2\rho} - K_1 \cdot \frac{3q\nu}{h^2} - \frac{h}{\rho}\frac{\partial p_s}{\partial x} + gh \cdot \left( \sin\theta - \cos\theta \frac{\partial h}{\partial x} \right) + \frac{\sigma h}{\rho}\frac{\partial^3 h}{\partial x^3} \tag{12}$$

where $K_1 = \frac{1 + \varepsilon/2}{1 + \varepsilon/4}$, $K_2 = \frac{1 + \varepsilon/3}{1 + \varepsilon/4}$.

Note that in undisturbed flow we can usually neglect the inertia term $\frac{\partial J}{\partial x}$ in the left part of (12), and this is justified at $\varepsilon \ll 1$. However, we will keep this component to apply the equation for film flow of liquid metals that have $\varepsilon \approx 1$. Using the velocity profile (9), we can calculate $J = \int_0^h u^2 \, dy$. After tedious transformations, we obtain

$$J = F_0 \cdot \frac{6 \cdot q^2}{5 \cdot h} + F_1 \cdot \frac{4 \cdot q \cdot h \cdot \tau_v}{5 \cdot \mu} + F_2 \cdot \frac{2 \cdot \tau_v^2 \cdot h^3}{15 \cdot \mu^2} \tag{13}$$

Here $F_0 = 1 - \frac{\varepsilon}{(\varepsilon + 4)^2}$, $F_1 = \frac{1 - \varepsilon/4}{(\varepsilon + 4)^2}$, $F_2 = \frac{1}{(\varepsilon + 4)^2}$.

Substituting $\tau_v = \tau_f + \frac{\varepsilon \cdot \mu \cdot V}{h}$ into (13), we separate in (13) the terms with parameter $\varepsilon$:

$$J = F_0 \frac{6q^2}{5h} + F_1 \frac{4qh\tau_f}{5\mu} + F_2 \frac{2\tau_f^2 h^3}{15\mu^2} + \frac{2\varepsilon V}{15} \left( 6F_1 q + 2F_2 h^2 \tau_f/\mu + \varepsilon \cdot F_2 h V \right) \tag{14}$$

The terms with $\varepsilon$ disappear if it is no phase transition. For immovable vapor we have in (14) only the first term.

Let us transform the kinematic condition (6) using the first equation from (1) to the form

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x} \int_0^h u \, dy = \frac{j}{\rho} \tag{15}$$

Substituting $j = \varepsilon \cdot \mu/h$ to the right part of (15), we rewrite it in the form

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = \frac{\varepsilon \cdot \nu}{h} \tag{16}$$

The set of equations (12) and (16) for functions $q(x,t)$, $h(x,t)$ describes the condensate film flow along an isothermal plane under action of gravity and turbulent vapor flow with a fixed velocity $V$.

## 4. Stationary (undisturbed) flow of condensate

Let us introduce the scale of film thickness $h_e = (\nu^2/g)^{1/3}$, the scale of velocity $u_e = (\nu g)^{1/3}$, dimensionless variables $h/h_e$, $q/\nu$, $x/h_e$. Keeping the previous notations for flow rate, thickness and coordinates, let us consider several specific cases of condensate stationary flow (when the surface tension is negligible compared to gravitation and impact from vapor stream). The impact of vapor on the film is taken into account through a given constant shear stress $\tau_f$ on the film surface.

### *4.1. Stagnant vapor. Flow of condensate on a vertical plate with regard for film inertia*

Eqs. (12) and (16) take the form

$$\begin{aligned} &F_0 \cdot \frac{6}{5} h^2 \frac{d}{dx}\left(\frac{q^2}{h}\right) = h^3 - 3K_1 q \\ &h\frac{dq}{dx} = \varepsilon \end{aligned} \tag{17}$$

System (17) has an analytical solution:

$$\begin{aligned} &q = h^3/3 \cdot (K_1 + 2F_0\varepsilon/3), \\ &h = \left( h^4(0) + 4 \cdot \varepsilon \cdot x \cdot (K_1 + 2F_0\varepsilon/3) \right)^{1/4} \end{aligned} \tag{18}$$

Unlike the classic solution by Nusselt [19] in (18), here the film inertia is taken into account through the coefficient $(K_1 + 2F_0\varepsilon/3)$, which differs from one.

### *4.2. Stagnant vapor. Flow on the bottom side of a horizontal plate with regard for film inertia.*

In this case the Eqs. (12) and (16) take the form

$$\begin{aligned} &F_0 \cdot \frac{6}{5} h^2 \frac{d}{dx}\left(\frac{q^2}{h}\right) = h^3\frac{dh}{dx} - 3K_1 q \\ &h\frac{dq}{dx} = \varepsilon \end{aligned} \tag{19}$$

Transforming additive $\frac{d}{dx}\left(\frac{q^2}{h}\right) = \frac{2q}{h}\frac{dq}{dx} - \frac{q^2}{h^2}\frac{dh}{dx} = \frac{2\varepsilon q}{h^2} - \frac{q^2}{h^2}\frac{dh}{dx}$ and excluding the variable $x$, we reduce the system (19) to equation

$$3h \cdot q \cdot (K_1 + 4F_0\varepsilon/5) \cdot \frac{dq}{dh} = \varepsilon \cdot \left( F_0 \cdot \frac{6}{5} q^2 + h^3 \right)$$

with solution

$$q^2 = \frac{2\varepsilon \cdot h^3}{9K_1 + 24F_0\varepsilon/5} - B \cdot h^N, \quad \text{where}$$
$$N = 4F_0\varepsilon/(5K_1 + 4F_0\varepsilon) < 1 \tag{20}$$

The constant $B$ we can pick up from condition $q = 0$ at $h = h(0)$. So we obtain

$$q = \sqrt{\frac{2\varepsilon \cdot h^3(0)}{9K_1 + 24F_0\varepsilon/5}}\left(\tilde{h}^3 - \tilde{h}^N\right) \tag{21}$$

where $\tilde{h} = h/h(0) \geqslant 1$ is the film thickness normalized to $h(0)$. Then we substitute (21) into the second equation of system (19) and integrate over parts: $\int h\,dq = [hq] - \int q\,dh = \varepsilon \int dx$, and obtain the dependency of the film thickness on variable $\tilde{x} = x/h(0)$:

$$\tilde{h}\sqrt{\tilde{h}^3 - \tilde{h}^N} - \int_1^{\tilde{h}} \sqrt{h'^3 - h'^N}\,dh' = 3 \cdot \tilde{x} \cdot \sqrt{\varepsilon \cdot (K_1 + 8F_0\varepsilon/15)/2h^3(0)} \tag{22}$$

In asymptotic case $\tilde{h} \gg 1$ we have $\tilde{h}\sqrt{\tilde{h}^3 - \tilde{h}^N} \approx \tilde{h}^{5/2}$, $\int_1^{\tilde{h}} \sqrt{h'^3 - h'^N}\,dh' \approx \frac{2}{5}\tilde{h}^{5/2}$. Then from (22) we obtain

$$\tilde{h} = \left(5 \cdot \tilde{x} \cdot \sqrt{\varepsilon \cdot (K_1 + 8F_0\varepsilon/15)/2h^3(0)}\right)^{2/5}$$

The graphs of film thickness calculated according to (22) at $h(0) = 1$ for different $\varepsilon$ are plotted in Fig. 2. The flow pattern is as follows: the film thickness is minimal in the center of the plate at $x = 0$ (while this, we have $\frac{dh}{dx}(0) = 0$, and velocity at the surface of film is $u_s(0) = 0$); it growths symmetrically towards the periphery, and the condensate spreads to the plate margins due to gravitation force.

*4.3. Stagnant vapor. Flow along the upper surface of horizontal plate with regard for film inertia*

Eq. (12) is as follows:

$$F_0 \cdot \frac{6}{5}h^2 \frac{d}{dx}\left(\frac{q^2}{h}\right) = -h^3 \frac{dh}{dx} - 3K_1 q$$

if we consider the second equation from the set (19), it can be transformed to the view $3h \cdot q \cdot (K_1 + 4F_0\varepsilon/5) \cdot \frac{dq}{dh} = \varepsilon \cdot \left(F_0 \cdot \frac{6}{5}q^2 - h^3\right)$. Its solution is

$$q = \sqrt{\frac{2\varepsilon \cdot h^3(0)}{9K_1 + 24F_0\varepsilon/5}\left(\tilde{h}^N - \tilde{h}^3\right)} \tag{23}$$

Here $\tilde{h} = h/h(0) \leqslant 1$; the symbols and boundary conditions are the same as in the previous case. Solution (23) exists at $\tilde{h}_* \leqslant \tilde{h} \leqslant 1$, where $\tilde{h}_* = (N/3)^{1/(3-N)}$ is the point of maximum $q(h)$, where $\frac{dq}{dh} \to 0$, while that $\frac{dh}{dx}|_{h=h_*} \to \infty$. The formula for film thickness as a function of variable $\tilde{x} = x/h(0)$ becomes the following:

$$\tilde{h}\sqrt{\tilde{h}^N - \tilde{h}^3} - \int_1^{\tilde{h}} \sqrt{h'^N - h'^3}\,dh' = 3 \cdot \tilde{x} \cdot \sqrt{\varepsilon \cdot (K_1 + 8F_0\varepsilon/15)/2h^3(0)} \tag{24}$$

If at $\varepsilon \ll 1$ in (20) we take $N = 0$, then $\tilde{h}^N = 1$. Substituting (23) into the second Eq. (19), we obtain the solution for film thickness:

$$\int_{\tilde{h}}^1 \frac{\xi^3}{\sqrt{1 - \xi^3}}\,d\xi = \sqrt{\frac{2\varepsilon}{h^3(0)}} \cdot \tilde{x}$$

This coincides (taking into account the scaling formalism) with the solution obtained in [20]. The dimensionless film thickness at the plate edges calculated according to (24) is determined unambiguously by value $\varepsilon, h(0)$, unlike [20] where it was a free parameter. The charts of film thickness, calculated according to (24) at

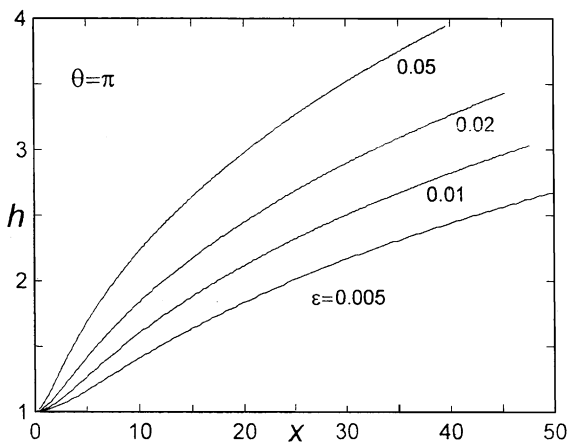


Fig. 2. Film thickness on the bottom side of horizontal plate.

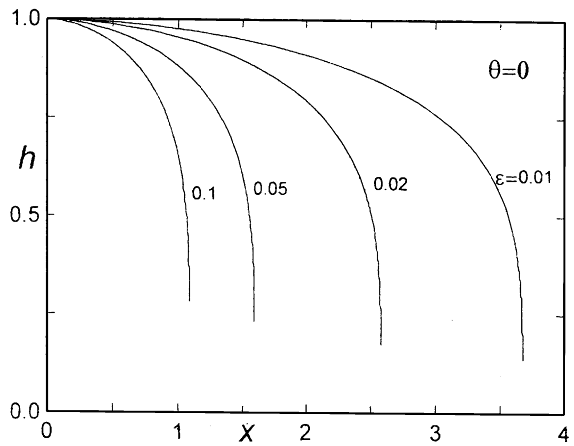


Fig. 3. Film thickness on the top side of horizontal plate.

$h(0) = 1$ for different $\varepsilon$ are plotted in Fig. 3. The flow pattern in this case is the following: the film thickness is maximal in the plate center (for $x = 0$) and decreases symmetrically towards the periphery. The condensate from the center spreads to periphery under gravity action and at $h/h(0) = (N/3)^{1/(3-N)}$, where $\frac{dh}{dx} \to \infty$, the film falls as a free jet from the plate edges.

### 4.4. Moving vapor. Flow without film inertia

For a regular liquid ($Pr \cong 1 - 10$) we can neglect the inertia term in the left part of (12) and the small term $\frac{dh}{dx}$ in the right part. We also take $K_1 = 1$, $K_2 = 1$ and bring the equation to the form

$$q = \frac{1}{2}(r_e \cdot h^2 + \varepsilon \cdot R_e h) + \frac{1}{3}h^3 \sin\theta \tag{25}$$

Here $R_e = V/u_e$, $r_e = \tau_f/\rho u_e^2 = C_f \frac{\rho_v}{2\rho} R_e \mid R_e \mid$.

Substituting (25) into (16) and integrating over $x$, we obtain the formula for film thickness as a function of coordinate:

$$\frac{1}{3}r_e \cdot h^3 + \frac{1}{4}(\varepsilon \cdot R_e h^2 + h^4 \sin\theta) = \varepsilon \cdot x \tag{26}$$

Here the integration constant was chosen to provide $h(0) = 0$. Figs. 4 and 5 present the dependency (26) for water vapor at $C_f = 3 \cdot 10^{-3}$, $\varepsilon = 0.01$ and different values of $R_e$ for cocurrent and countercurrent flow of vapor. In case $V > 0$ the direction of vapor flow and liquid flow coincides with the direction of gravitation ($R_e > 0, r_e > 0$), and film thickness increases along the axis $Ox$. The asymptotic solutions of (26) are:

$$\begin{aligned} x \to 0 \quad & h(x) = \sqrt{4 \cdot x/R_e} \\ x \to \infty \quad & h(x) = (4\varepsilon \cdot x/\sin\theta)^{1/4} \end{aligned} \tag{27}$$

For the case of $V < 0$ when the vapor flow direction is opposite to gravity ($R_e < 0, r_e < 0$), Eq. (26) gives two branches of $h(x)$ at $x < 0$. For the low branch $h_l(x)$, condensate moves upwards the plane overcoming gravity action due to vapor flow. The film thickness increases in the same direction and at point of branching $x = x^*$ it reaches the top level $h^* = \left(\mid r_e \mid + \sqrt{r_e^2 + 2\varepsilon \mid R_e \mid \sin\theta}\right)/2\sin\theta$. While that, $\frac{dh}{dx}|_{x=x_*} \to \infty$. For the upper branch $h_{up}(x) > h^*$ and the film thickness increases with coordinate $x$.

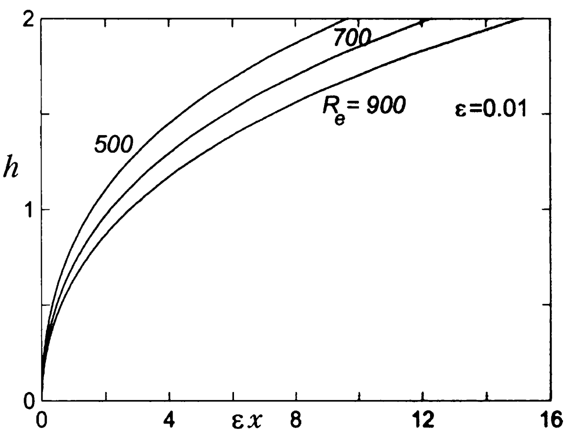


Fig. 4. Film thickness on the vertical plate (cocurrent flow).

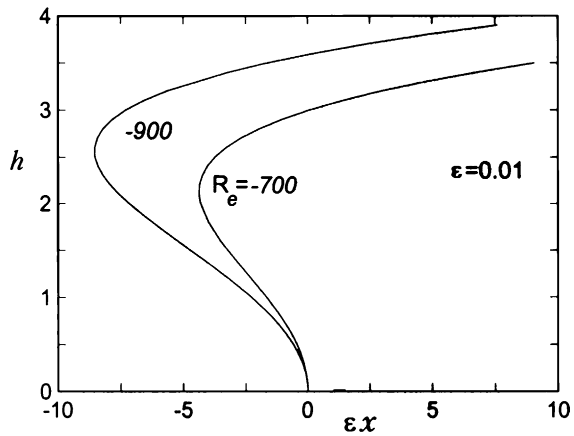


Fig. 5. Film thickness on the vertical plate (countercurrent flow).

It follows from (25) that $q > 0$ at $h > h_+$, and $q < 0$ at $h < h_+$. Here $h_+ = \frac{3}{4\sin\theta}\left(\mid r_e \mid + \sqrt{r_e^2 + \frac{8}{3}\varepsilon \mid R_e \mid \sin\theta}\right) > h_{up}(0)$. Thus for the upper branch and the range $h^* < h_{up}(x) < h_+$ the condensate moves upward driven by the vapor flow, but for range $h_{up}(x) > h_+$ gravitation dominates and the condensate is driven downwards.

## 5. Stability of stationary condensate film flow

The use of a linear approximation for the temperature profile in the film for stability analysis indicates that the typical disturbance frequency $\omega$ satisfies the condition $\omega \ll \lambda/(\rho c_p h^2)$, i.e., the temperature profile has the time to adjust to the disturbed film thickness.

For analysis of flow stability, we can choose the scales size equal to the film thickness $h_m$ at the coordinate $x$, where stability is studied. Let us introduce the scales for velocity $u_m = gh_m^2/3\nu$, time $t_m = h_m/u_m$, flow rate $q_m = h_m u_m$, stress $p_m = \rho g h_m/3$, and make up the dimensionless variables $x/h_m$, $u/u_m$, $q/q_m$, $t/t_m$, $h/h_m$ $p_s/p_m$, $J/u_m^2 h_m$, keeping the old symbols. Here we assume that Prandtl number $Pr \cong 1 - 10$ (regular liquid), so at $Ku \gg 1$ the parameter $\varepsilon \ll 1$ and $K_1 = 1, K_2 = 1$. In dimensionless variables the Eqs. (12) and (16) take the form

$$\frac{\partial q}{\partial t} + \frac{\partial J}{\partial x} = \frac{3}{Re}\left(h \cdot \left(\sin\theta - \cos\theta\frac{\partial h}{\partial x}\right) + \frac{1}{2}\left(\tau + \frac{\varepsilon R_v}{h}\right) - \frac{q}{h^2} - \frac{h}{3}\frac{\partial p_s}{\partial x}\right) + We \cdot h \cdot \frac{\partial^3 h}{\partial x^3} \tag{28}$$

$$\frac{\partial h}{\partial t}+\frac{\partial q}{\partial x}=\frac{\varepsilon}{Re\cdot h} \tag{29}$$

Here $Re = gh_m^3/3\nu^2$ is the Reynolds number, $We = \sigma/\rho h_m u_m^2 = (3Fi)^{1/3}/Re^{5/3}$ is Weber number, $Fi = \sigma^3/\rho^3 g\nu^4$ is the film number, $\tau = 3\cdot\tau_f/\rho g h_m$ is the dimensionless shear stress, $R_v = V/u_m = R_m/Re^{2/3}$ and $R_m = V(3/\nu g)^{1/3}$ are dimensionless vapor velocities,

$$J=\frac{6\cdot q^2}{5\cdot h}+\frac{q\cdot h\cdot\tau}{20}+\frac{\tau^2\cdot h^3}{120}+\frac{\varepsilon\cdot R_v}{20}\left(q+\frac{h^2\tau}{3}+\frac{\varepsilon\cdot R_v\cdot h}{6}\right).$$

For undisturbed flow, the system of equations (28) and (29) for variables $h_0(x)$, $q_0(x)$ takes the form:

$$q_0=\frac{1}{2}(r\cdot h_0^2+\varepsilon\cdot R_v h_0)+h_0^3\sin\theta$$
$$\frac{dq_0}{dx}=\frac{\varepsilon}{Re\cdot h_0} \tag{30}$$

Here subscript "0" means the undisturbed state, $r = 3\tau_{f0}/\rho g h_m = r_m/Re^{1/3}$, $r_m = C_f\rho_v R_m|R_m|/2\rho$ is the dimensionless shear stress on the interface. During formulation of $q_0$ in (28), the terms with derivatives $\frac{dJ_0}{dx}$, $\frac{dh_0}{dx}$, $\frac{d^3h_0}{dx^3}$ were discarded, since they have infinitesimal order $O(\varepsilon), O(\varepsilon), O(\varepsilon^3)$. This means that at $\varepsilon \ll 1$ the film inertia and surface tension are negligible in comparison with gravitation and friction. The remained terms have the infinitesimal order O(1).

The term $\varepsilon R_v/h$ was kept because for a typical vapor velocity $V \cong 10$m/c we have $R_m \cong V(3/\nu g)^{1/3} \cong 10^3$, so even at $\varepsilon \cong 10^{-3} \ll 1$ the parameter $\varepsilon R_v \cong 1$. Taking the first of equations from (30) and putting it into the second provides us the following:

$$\frac{dh_0}{dx}=\frac{\varepsilon}{Re\cdot h_0(3h_0^2\sin\theta+r\cdot h_0+\varepsilon\cdot R_v/2)} \tag{31}$$

We assume that we study stability far away from the values of $h_0(x)$ which make the divider in (31) turn into zero (at $h_0 = 0$ and at $h_0 = \left(|r|+\sqrt{r^2+6\varepsilon|R_v|\sin\theta}\right)/6\sin\theta$ for the countercurrent flow).

### 5.1. Equations for small disturbances of stationary flow

Let us consider the linearization (28), (29) relative to small disturbances of stationary flow, assuming decomposition $h(x,t) = h_0\cdot(1 + H(x,t)), q(x,t) = q_0 + Q(x,t)$, $\tau = r\cdot(1+\tau'), p_s = p_{s0} + r\cdot p'$, where $H$, $Q$, $\tau'$, $p'$ are small disturbances, and undisturbed parameters $h_0, q_0 \approx O(1)$. After linearization we obtain

$$\frac{\partial Q}{\partial t}+\frac{\partial\Delta J}{\partial x}=\frac{3}{Re}\left(\left(h_0\sin\theta+\frac{2q_0}{h_0^2}-\frac{\varepsilon R_v}{2h_0}\right)H-\frac{Q}{h_0^2}+\frac{r\tau'}{2}-\frac{rh_0}{3}\frac{\partial p'}{\partial x}-h_0^2\cos\theta\frac{\partial H}{\partial x}\right)+We h_0^2\frac{\partial^3 H}{\partial x^3}$$
$$h_0\frac{\partial H}{\partial t}+\frac{\partial Q}{\partial x}=-\frac{\varepsilon\cdot H}{Re\cdot h_0} \tag{32}$$

In (32) we also discarded the terms with derivatives $\frac{dh_0}{dx}$, $\frac{d^3h_0}{dx^3}$ having the order $O(\varepsilon)$, $O(\varepsilon^3)$. Here $\Delta J$ is the linear part of disturbance for value $J$:

$$\Delta J=\left(\frac{\partial J}{\partial h}\right)_0 h_0\cdot H+\left(\frac{\partial J}{\partial q}\right)_0 Q+\left(\frac{\partial J}{\partial\tau}\right)_0 r\cdot\tau' = -a_1\cdot H+2a_2\cdot Q+a_3\cdot r\cdot\tau'$$

$$a_1=-\left(\frac{\partial J}{\partial h}\right)_0 h_0=\frac{6q_0^2}{5h_0}-\frac{q_0 r h_0}{20}-\frac{r^2h_0^3}{40}-\frac{\varepsilon R_v h_0}{30}\left(r\cdot h_0+\frac{\varepsilon\cdot R_v}{4}\right)$$
$$2a_2=\left(\frac{\partial J}{\partial q}\right)_0=\frac{12q_0}{5h_0}+\frac{1}{20}(r\cdot h_0+\varepsilon\cdot R_v)$$
$$a_3=\left(\frac{\partial J}{\partial\tau}\right)_0=\frac{q_0h_0}{20}+\frac{h_0^2}{60}(r\cdot h_0+\varepsilon\cdot R_v) \tag{33}$$

Unlike the case of film without phase transition, here we have coefficients $a_1$, $a_2$, $a_3$ depending on coordinate $x$. Calculating the derivative $\frac{\partial\Delta J}{\partial x}$ in the left part of Eq. (32), we obtain

$$\frac{\partial\Delta J}{\partial x}=-a_1\frac{\partial H}{\partial x}+2a_2\frac{\partial Q}{\partial x}+a_3\cdot r\cdot\frac{\partial\tau'}{\partial x}-H\frac{da_1}{dx}+2Q\frac{da_2}{dx}+\tau'\cdot r\cdot\frac{da_3}{dx} \tag{34}$$

Derivatives from coefficients $a_1$, $a_2$, $a_3$ over coordinate $x$, found with the second equation from (30): $\frac{da_j}{dx}=\frac{da_j}{dq_0}\cdot\frac{dq_0}{dx}=\left(\frac{\partial a_j}{\partial q_0}+\frac{\partial a_j}{\partial h_0}\frac{dh_0}{dq_0}\right)\frac{\varepsilon}{Re\,h_0}$, $j = 1,2,3$ have the order $O(\varepsilon)$ and we can disregard them. Here $\frac{dh_0}{dq_0}=\frac{1}{(3h_0^2\sin\theta+r\cdot h_0+\varepsilon\cdot R_v/2)}\cong O(1)$.

Substituting (34) into (32), we write down the equations in the form

$$\frac{\partial Q}{\partial t}-a_1\frac{\partial H}{\partial x}+2a_2\frac{\partial Q}{\partial x}+a_3 r\frac{\partial\tau'}{\partial x}=\frac{3}{Re}\left(H\cdot A-\frac{Q}{h_0^2}+\frac{r\tau'}{2}-\frac{rh_0}{3}\cdot\frac{\partial p'}{\partial x}-h_0^2\cos\theta\frac{\partial H}{\partial x}\right)+We h_0^2\frac{\partial^3 H}{\partial x^3}$$
$$h_0\frac{\partial H}{\partial t}+\frac{\partial Q}{\partial x}=-\frac{\varepsilon\cdot H}{Re\cdot h_0} \tag{35}$$

Here

$$A=h_0\sin\theta+2q_0/h_0^2-\varepsilon\cdot R_v/2h_0 \tag{36}$$

Substituting the relations of quasilaminar model [10] into the system (35), we obtain formulas linking the disturbances in the shear stress with the disturbances in film thickness:

$$p'=p_R H+\frac{p_I}{k}\frac{\partial H}{\partial x},\quad \tau'=\tau_R H+\frac{\tau_I}{k}\frac{\partial H}{\partial x}$$

where $k$ is the wavenumber, $p_R$, $\tau_R$, $p_I$, $\tau_I$ are the real and imaginary parts of complex amplitude of stress.

Collecting the similar terms, we bring the equations (35) to the form

$$\frac{\partial Q}{\partial t}+2a_2\frac{\partial Q}{\partial x}+\frac{3}{h_0^2 Re}\cdot Q=\frac{3\cdot c_0}{Re}H+b\frac{\partial H}{\partial x}+n_0\frac{\partial^2 H}{\partial x^2}+We\cdot h_0^2\frac{\partial^3 H}{\partial x^3}$$
$$h_0\frac{\partial H}{\partial t}+\frac{\partial Q}{\partial x}=-\frac{\varepsilon\cdot H}{Re\cdot h_0} \quad (37)$$

Here

$$c_0=A+r\cdot\tau_R/2,\quad n_0=-r\cdot(a_3\tau_I/k+p_I h_0/kRe)$$
$$b=a_1-\frac{3\cos\theta\cdot h_0^2}{Re}+r\cdot\left(\frac{3\tau_I/2k-p_R h_0}{Re}-a_3\tau_R\right) \quad (38)$$

### 5.2. Two-wave equation

Let us find the solution of (37) in the form

$$H=H_a\exp(\mathrm{i}k(x-ct)+\beta t),$$
$$Q=Q_a\exp(\mathrm{i}k(x-ct)+\beta t) \quad (39)$$

where $k$ is the wavenumber, $c(k)$, $\beta(k)$ are the phase velocity and wave increment, $H_a$, $Q_a$ are the amplitudes of disturbances. First we have to consider the asymptotic solutions at $\frac{dh_0}{dx}\ll k\ll 1$. This means that the wavelength of disturbance is much less than the distance where the film thickness changes (but still much higher than the film thickness). In this case we take $h_0$ as a slow-varying function, so during differentiation over $x$ for the film thickness and coefficients $a_1$, $a_2$, $a_3$, $b$, $c_0$, $c_1$, $c_2$, $n_0$ can be taken as constants (the rejected terms have the order O($\varepsilon$)). Differentiation of first equation from (37) over $x$, and second-over $t$ produces the following:

$$\frac{\partial^2 Q}{\partial t\partial x}+2a_2\frac{\partial^2 Q}{\partial x^2}+\frac{3}{Re h_0^2}\frac{\partial Q}{\partial x}=\frac{3c_0}{Re}\frac{\partial H}{\partial x}+b\frac{\partial^2 H}{\partial x^2}+n_0\frac{\partial^3 H}{\partial x^3}+We h_0^2\frac{\partial^4 H}{\partial x^4}$$
$$h_0\frac{\partial^2 H}{\partial t^2}+\frac{\varepsilon}{Re}\frac{\partial H}{\partial t}=-\frac{\partial^2 Q}{\partial x\partial t} \quad (40)$$

Excluding from (40) $Q(x,t)$ and neglecting the small terms with order O($\varepsilon$)), we obtain one equation for disturbance of film thickness:

$$h_0\left(\frac{\partial}{\partial t}+c_1\frac{\partial}{\partial x}\right)\left(\frac{\partial}{\partial t}+c_2\frac{\partial}{\partial x}\right)H+\frac{3}{Re h_0}\left(\frac{\partial H}{\partial t}+c_0 h_0\frac{\partial H}{\partial x}\right)+n_0\frac{\partial^3 H}{\partial x^3}+We h_0^2\frac{\partial^4 H}{\partial x^4}=0 \quad (41)$$

Here $c_{1,2}=a_2\pm\sqrt{a_2^2-b/h_0}$.

The two-wave equation for a film without phase transition was obtained in [15]. The first wave operator in (41) describes the dynamic waves moving with velocities $c_1$ and $c_2$ which makes the main contribution at high $Re$. The second wave operator describes a kinematic wave moving with velocity$c_0h_0$, which makes the main contribution at low $Re$. Unlike the case $\varepsilon = 0$, the velocities $c_0, c_1, c_2$ are slow-varying function of coordinate $x$. As parameter $Re$ increases, the relative contribution of kinematic wave reduces proportionally to $\frac{1}{Re^{5/3}}$.

### 5.3. Dispersion relations

The flow stability has a local nature. At a given vapor velocity and temperature load (given $R_m$, $\varepsilon$), the change in coordinate brings the change in local $Re$ and, correspondingly, in parameters $R_v, r$, We, that are included to Eq. (37). Taking into account the chosen scale $h_m$, in (37), (38), (36), (33), (30) we assume $h_0 = 1$. Substitution of (39) into (37) brings out the dispersion equation

$$\left(\beta+\frac{3}{Re}+\mathrm{i}k(2a_2-c)\right)\left(\beta+\frac{\varepsilon}{Re}-\mathrm{i}kc\right)=-\mathrm{i}k\left(\frac{3c_0}{Re}-n_0k^2+\mathrm{i}k(b-We\cdot k^2)\right) \quad (42)$$

It follows from (42) that at $k = 0$ $\beta(0) = -\varepsilon/Re$ or $\beta(0) = -3/Re$, i.e., the long-wave disturbances are stable for both modes, unlike the case of film without condensation. Taking separately out from (42) its real and imaginary parts, we obtain the dispersion relations in the form:

$$\frac{\beta Re}{3}+\frac{1+\varepsilon/3}{2}=-\frac{A_0}{2(a_2-c)}$$
$$\left(\frac{\beta Re}{3}+\frac{1+\varepsilon/3}{2}\right)^2=\left(\frac{kRe}{3}\right)^2((c-a_2)^2-B_0)+\left(\frac{1-\varepsilon/3}{2}\right)^2 \quad (43)$$

Here $A_0 = c_0 - a_2(1 - \varepsilon/3) - n_0k^2Re/3$, $B_0 = a_2^2 - b + We\cdot k^2$.

Excluding from (43) the increment $\beta$, we obtain for $\chi = (c - a_2)^2$ the square equation:

$\chi^2 - \chi\cdot(B_0 - S\cdot(1 - \varepsilon/3)^2) - A_0^2\cdot S = 0$, where $S = (3/2kRe)^2$, that gives us

$$\chi=\left(B_0-S\cdot(1-\varepsilon/3)^2+\sqrt{(B_0-S(1-\varepsilon/3)^2)+4A_0^2\cdot S}\right)/2$$
$$c=a_2\pm\sqrt{\chi},\quad \frac{\beta Re}{3}=-\left(\frac{1+\varepsilon/3}{2}\right)\pm\frac{A_0}{2\sqrt{\chi}} \quad (44)$$

The dispersion relations give us two wave modes that have correspondence to signs (+,−) in formulas (44). The first sign (sign "+") means that the main mode may be unstable (positive increment), and the second mode is stable always. For a film with condensation, the dispersion relations can be solved analytically (same as in [16]). All formulas at $\varepsilon = 0$ transform into expressions derived previously for a film without phase transition. Note that relations (43) are valid even for the case of evaporation instead of condensation, i.e., for $\varepsilon < 0$. However, it would be wrong to assume that the effect of evaporation is always opposite to the effect of condensation—because parameter $\varepsilon$ gives nonlinear contribution into dispersion relationships. Although accepted model is applicable for $\varepsilon < 0$, the stability of evaporating film requires special consideration and this is beyond the sphere of this paper.

### 5.4. Neutral stability curve for stagnant vapor

Let us consider the condition $\beta = 0$ for neutral waves with V = 0 (stagnant vapor). It follows from (43):

$$\frac{(1+\varepsilon/3)^2}{4} = \left(\frac{k_n Re}{3}\right)^2 \left(\left(\frac{A_0}{1+\varepsilon/3}\right)^2 - a_2^2 + b - Wek_n^2\right) + \frac{(1-\varepsilon/3)^2}{4}$$

$$c = a_2 + \frac{A_0}{1+\varepsilon/3} \tag{45}$$

Let us transform (45) into a biquadratic equation relative to the neutral wave number $k_n$:

$$(3Fi)^{1/3}k_n^4 - MRe^{5/3}k_n^2 + \frac{3\cdot\varepsilon}{Re^{1/3}} = 0$$

where $M = \left(\frac{A_0}{1+\varepsilon/3}\right)^2 - a_2^2 + b$. The solution is

$$k_n = \left(\left(MRe^{5/3} \pm \sqrt{M^2Re^{10/3} - 12\cdot(3Fi/Re)^{1/3}\varepsilon}\right)\frac{1}{2(3Fi)^{1/3}}\right)^{1/2} \tag{46}$$

The signs "+" and "−" in (46) correspond to upper and lower branches of neutral curve. The critical Reynolds number $Re_c$, which marks the branching of the curve is found from the condition $M^2\cdot(Re_c^{11}/3Fi)^{1/3} = 12\cdot\varepsilon$. For $Re < Re_c$ the film flow is stable for all values of wave numbers. At $\varepsilon \ll 1$ we obtain $M \approx 3\cdot(\sin^2\theta - \frac{\cos\theta}{Re_c})$. So $Re_c$ is found by the equation

$$Re_c \sin\theta = ctg\theta + \frac{2}{\sqrt{3}(Re_c\sin\theta)^{5/6}}\left(\varepsilon\cdot\left(\frac{3Fi}{\sin\theta}\right)^{1/3}\right)^{1/2} \tag{47}$$

Eq. (47) may have one, two or three roots (depending on parameter $\varepsilon$), but only the biggest root has a physical meaning, because only for it $k_n^2 > 0$. If there is no phase transition, it follows from (47) that $Re_c \sin\theta = ctg\theta$. For a vertical film we obtain from (47) $Re_c = (64Fi\varepsilon^3/9)^{1/11}$. Relation (47) gives the numerical multipliers before the right-part terms—they are slightly different from those obtained in [6,9]. In particular, the critical Reynolds number from [6] for a vertical film (note the scaling factors) is different from the result of (47) by factor of 0.90. The formula derived from the Orr–Sommerfeld equation [9] gives the $Re_c$ higher than (47) by factor of 1.22. The difference must be due to assigned parabolic velocity profile in the integral method.

## 6. Calculation results

Here we present the calculations for dispersion relations (44) for water vapor at $T_s = 373$ K, $Fi^{1/3} = 9800$, $C_f = 3\cdot10^{-3}$ ($C_f$ is taken from [21]). Fig. 6 presents the dependency of the critical Reynolds number (47) on the inclination angle $\theta$ at different $\varepsilon$. The significant dependency $Re_c(\theta)$ is explicit only at $\theta \approx 0$ and $\theta \approx \pi$. For the film on the top surface ($\theta \approx 0$) the values of $Re_c$ are much higher than for the same film at the back surface ($\theta \approx \pi$), with gravitation producing the Rayleigh–Taylor instability.

### 6.1. Vertical film

The neutral curves are plotted in Fig. 7 for the case of stagnant vapor at different values of $\varepsilon$. To the left of this curve the flow is stable, and it is unstable at the right. With growth of parameter $\varepsilon$ the instability zone shrinks (due to the long-wave boundary), i.e., condensation of stagnant vapor stabilizes the film flow. At high $Re$ the neutral curves asymtotically approaches the appropriate curve for the no-phase-transition film: at $Re \gg Re_c$ condensation of stagnant vapor weakly influence the stability of film flow. For the moving flow, effect of condensation on film stability depends on direction of vapor flow.

The dispersion curves $\beta(k)$, $c(k)$ for cocurrent vapor flow at $r_m = 0.5$ are plotted in Fig. 8. The increase in parameter $\varepsilon$ brings the growth of phase velocity and wave increment (with the exception of very low $k$). For the latter case, the growth of $\varepsilon$ increases the positive contribution of condensation into the shear stress on the interface; the shear stress, as well as gravitation, increases

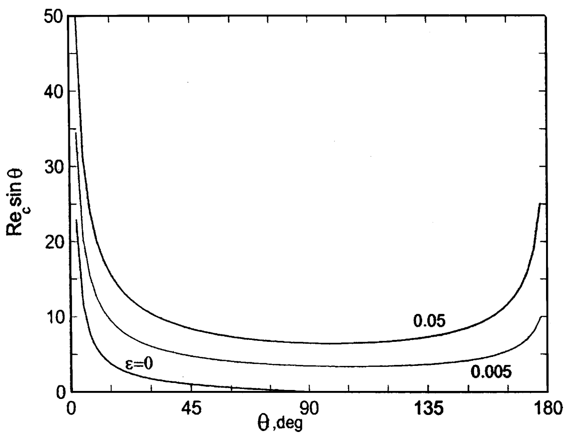


Fig. 6. Critical Reynolds number upon the inclination angle of the plate.

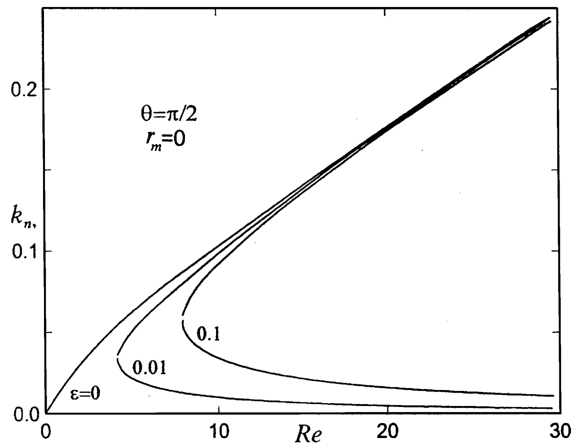


Fig. 7. Neutral curves for the vertical film (stagnant vapor).

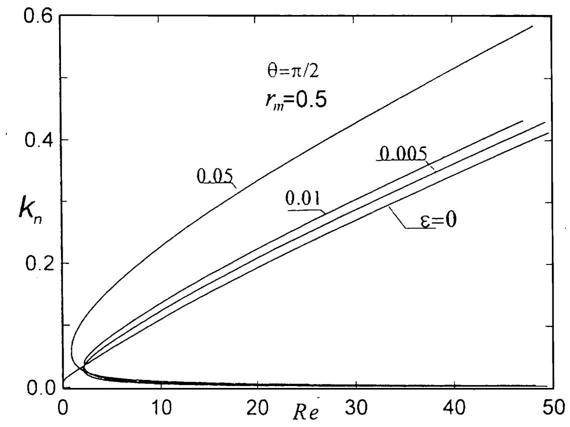


Fig. 9. Neutral curves for vertical film (cocurrent flow).

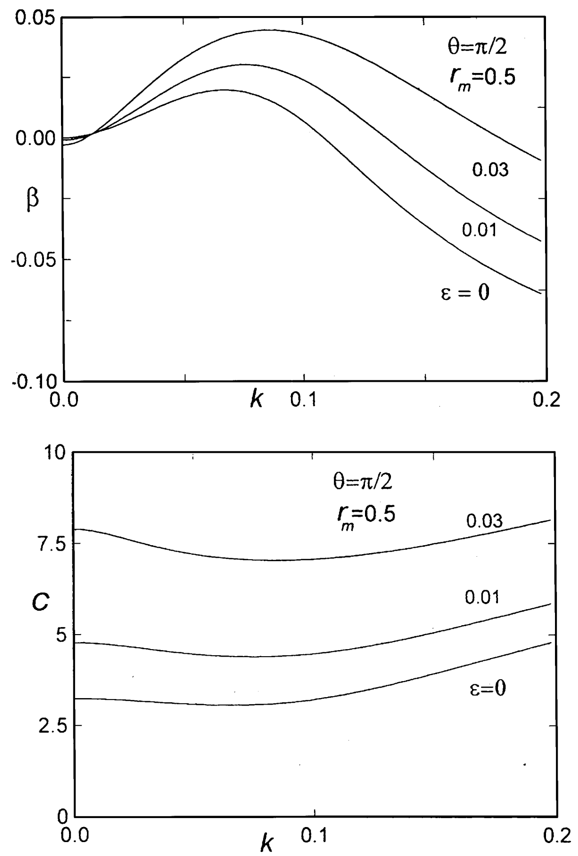


Fig. 8. Dispersion curves for the vertical film (cocurrent flow).

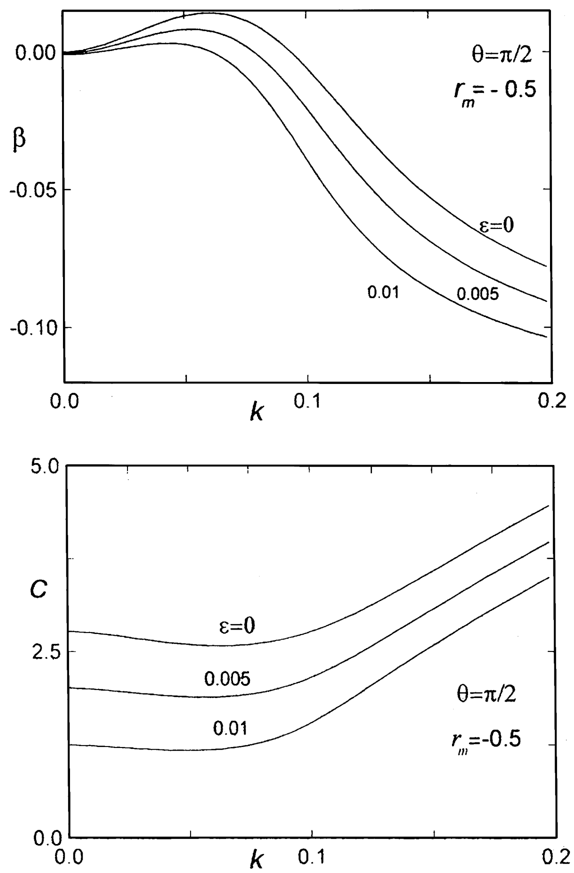


Fig. 10. Dispersion curves for the vertical film (countercurrent flow).

the kinetic energy of film and feed up the small disturbances. Corresponding neutral curves are shown in Fig. 9. The growth in ε brings out (for $\varepsilon \geqslant 10^{-2}$) to expansion of instability zone (with the exception of the coordinate origin). This effect of expansion of instability zone for cocurrent gas-film flow (no phase transition) had been marked out in [13,16]. Thus, for cocurrent flow, the condensation from vapor reduces the film stability.

For countercurrent vapor flow, condensation produces the opposite effect. The dispersion curve at

$r_m = -0.5$ is shown in Fig. 10. The growth in ε decreases the phase velocity and wave increment. For this case, the range of instability of wave number is narrower than for the cocurrent flow. The corresponding neutral curves are plotted in Fig. 11. An increase in ε makes the instability zone narrower. The effect of condensation parameter on the critical Reynolds number is plotted in Fig. 12 for different $r_m$. For the countercurrent vapor flow the dependency of $Re_c(\varepsilon)$ is monotonic, unlike the case of cocurrent vapor flow. Thus, usually the countercurrent flow of condensate film and vapor is more stable than the cocurrent flow at the same $|r_m|$, $Re$. Note that for the studied range of Re the neutral curve for moving vapor (the same as for the cocurrent and for the countercurrent flows) is very different from the neutral curve for film without phase transition. This means that stability of condensate film flow depends more on film interaction with the turbulent vapor flow (and condensation parameters), and less—on gravity. Figs. 13 and 14 present the plottings of dimensionless velocity $c_{max}/(\nu g)^{1/3}$ for the maximal-growth wave as a function of $Re$ both for countercurrent and cocurrent vapor flow at different values of ε. For the cocurrent flow, this velocity increases with ε, but declines with ε for countercurrent flow.

### 6.2. Horizontal film

The dispersion curves at $r_m = 0.5$ for the film beneath the horizontal plane is plotted in Fig. 15. The values of phase velocity have a weak dependency on $k$ at $k < 0.1$, but depend significantly on parameter ε. Neutral curves for this case are shown in Fig. 16. With the growth of condensation parameter ε, the instability zone first shrinks, but then it becomes larger, and $Re_c$ reduces. The corresponding dependencies of $Re_c(\varepsilon)$ for different $r_m$ are plotted in Fig. 17. At low $r_m$, the main contribution

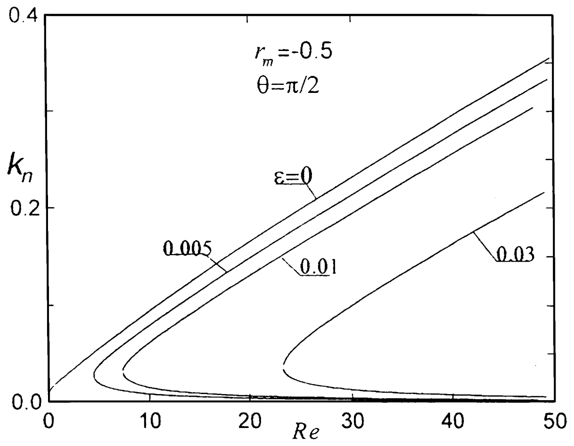


Fig. 11. Neutral curves for the vertical film (countercurrent flow).

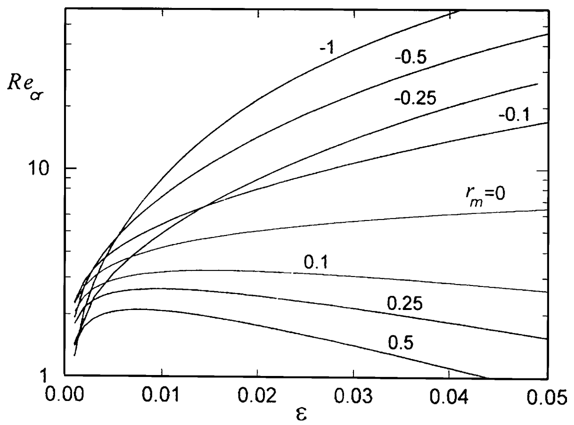


Fig. 12. Critical Reynolds number for the vertical film.

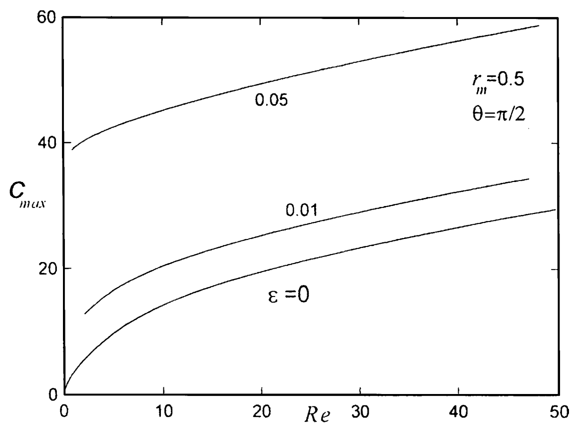


Fig. 13. Velocity of maximum growth wave (cocurrent flow).

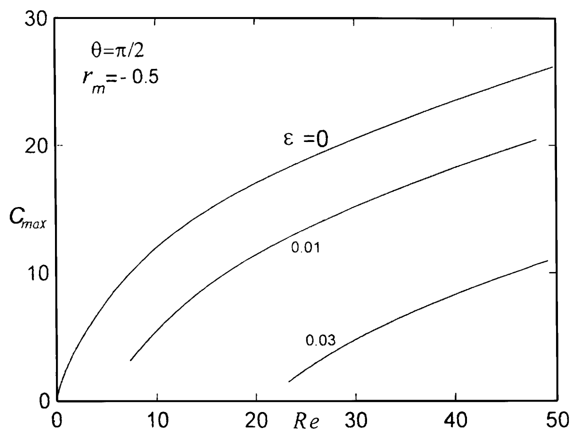


Fig. 14. Velocity of maximum growth wave (countercurrent flow).

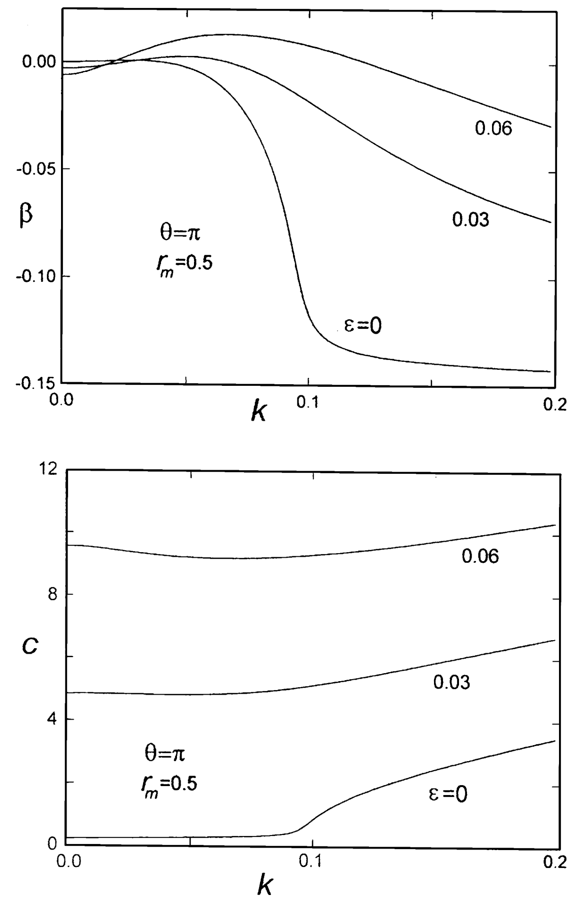


Fig. 15. Dispersion curves for the film on the bottom side of horizontal plate.

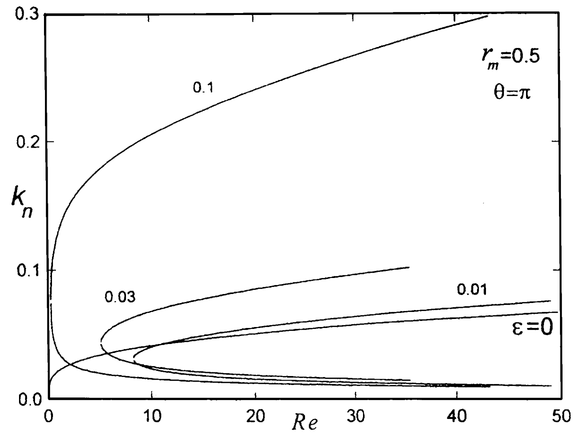


Fig. 16. Neutral curves for the film on the bottom side of horizontal plate.

to film instability is made by the Rayleigh–Taylor instability, and phase transition stabilizes film flow. With a

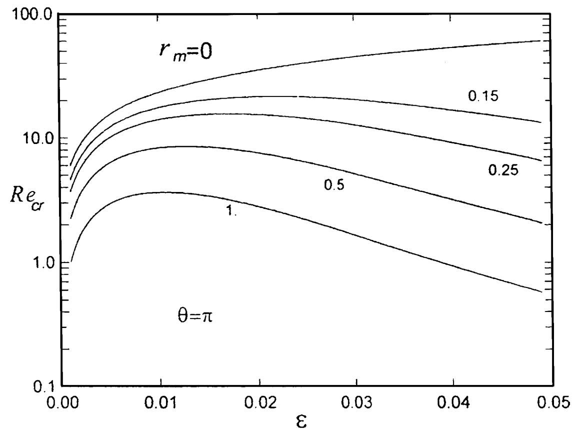


Fig. 17. Critical Reynolds number for the film on the bottom side of horizontal plate.

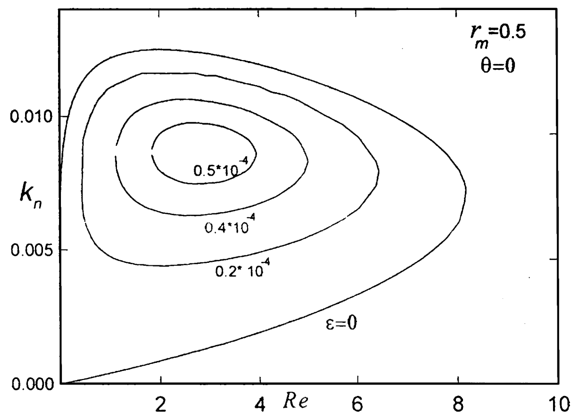


Fig. 18. Instability domain for the film on the top side of horizontal plate at low $Re$.

higher vapor velocity, the instability of film is governed by its interaction with the moving vapor. As parameter $\varepsilon$ increases, condensation gives destabilization effect due to a higher shear stress on the interface.

The effect of condensation on stability of a film flowing on the top side of a plane is quite peculiar. The neutral curves at $r_m = 0.5$ are plotted in Figs. 18–20. Without phase transition, at low $r_m$ the instability zone looks like non-connected domains (two separate domains). The first domain is a close-end patch at low $Re$ (Fig. 18), the second one is restricted by a open-end curve (high $Re$, Fig. 19). With the growth of condensation parameter $\varepsilon$, the first domain shrinks and disappears completely at $\varepsilon \approx 0.57 \cdot 10^{-4}$, but the second domain expands as a "tongue", stretched to the side of low $Re$. For high values of $r_m$, the domain of instability forms one zone (Fig. 20). With the growth of parameter $\varepsilon$ it shrinks drastically, but then at $\varepsilon \geqslant 10^{-2}$ it expands to the side of small values of $Re$.

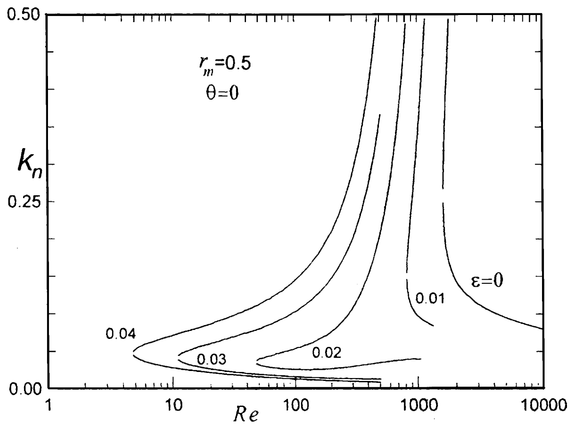


Fig. 19. Instability domain for the film on the top side of horizontal plate at high $Re$.

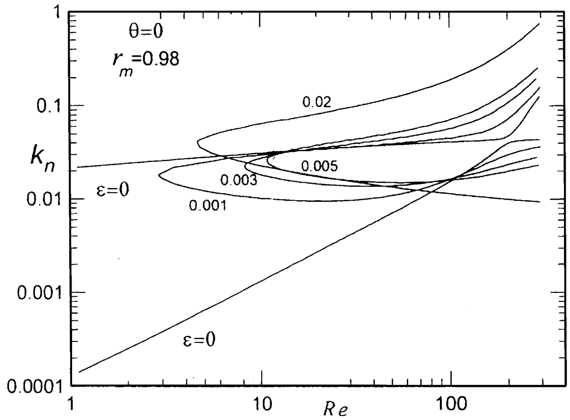


Fig. 20. Instability zone at high $r_m$.

## 7. Conclusions

1. Both for moving and stagnant vapor the process of condensation increases the stability of film flow in the long-wave region.
2. Condensation stabilizes the film flow only for the case of stagnant flow for all values of angle $\theta$ and for the countercurrent vapor flow. For the rest of cases, condensation effect on film stability is unambiguous. For low values of condensation parameter $\varepsilon$ the instability domain shrinks and the value of $Re_c$ becomes higher; the "thin" film (low $Re$) becomes stable to disturbances with any wavelength. The further growth of $\varepsilon$ makes the instability domain wider and it shifts to low $Re$, and for a "thick" film (high $Re$) the condensation always expands the range of wave number corresponding to instability.
3. The countercurrent flow of vertical condensate film and vapor flow at $|r_m| \approx 1$ is more stable than the cocurrent flow at the same values of $|r_m|$, $Re$.
4. With the vapor velocity increases, the impact of moving vapor and condensation becomes the key factors contributing to film instability.

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